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CALCULATION OF HEAT TRANSFER RATES TO GAS TURBINE BLADES

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Abstract—Five computer programs are used to calculate heat transfer rates to gas turbine blades and the results compared with experimental work on blade cascades. The programs use different methods to model the turbulence quantities. The experimental work was carried out recently in the University of Oxford free-piston tunnel.

NOMENCLATURE

- b, blade chord;
- C_{ax}, axial chord;
- e, turbulence mixing energy;
- k, thermal conductivity;
- P, free-stream pressure;
- Pr, Prandtl number;
- q, Q, heat flux;
- Re, Reynolds number at inlet to cascade $= U_1 b/\nu$;
- R_e^-, low Reynolds number flow (see Table 2);
- R_e^+, high Reynolds number flow (see Table 2);
- Re_θ, Reynolds number based on the momentum thickness $= \delta_\theta U_1/\nu$;
- S, length of suction surface or pressure surface measured from the stagnation point;
- T, local mean temperature;
- T_u, free-stream turbulence level;
- U, local mean velocity in streamwise direction;
- U_τ, friction velocity $= (\tau_w/\rho)^{1/2}$;
- x, coordinate for specifying blade profile (see Table 1);
- X, distance along suction or pressure surface from stagnation point;
- X^+, dimensionless distance downstream from the start of heat transfer $= \int_0^x U_\tau/\nu \, dX$;
- y, coordinate for specifying blade profile (see Table 1);
- Y, distance from the blade surface;
- Y^+, dimensionless distance from blade surface $= YU_\tau/\nu$;
- δ_θ, momentum thickness $= \int_0^\infty \rho U/\rho_1 U_1 (U_1 - U)/U_1 \, dy$;
- ρ, density;
- μ, absolute viscosity;
- ν, kinematic viscosity;
- τ, shear stress;
- ω, turbulence dissipation rate.

Subscripts

- t, turbulent value;
- T, stagnation value;
- w, value at wall;
- 1, free-stream value.

1. INTRODUCTION

A KNOWLEDGE of the heat fluxes to and from the external and internal surfaces of gas turbine blading is an essential input into the design of the blade cooling system. The heat fluxes form the boundary conditions in the conduction analysis necessary to predict the blade metal temperatures. A detailed discussion is contained in Daniels [1]. The blade metal temperatures may of course be obtained experimentally. Such a course of action is very expensive, and too time consuming to be of use in the iterative design and optimisation process. A computer program capable of giving accurate predictions of the surface heat fluxes to gas turbine blades would be of great assistance to engine designers and should also help to reduce the amount and thus cost of development testing required in an engine design.

The prediction of the development of the boundary layer on a gas turbine will inevitably be very difficult. The highly accelerating nature of the flow, the high freestream disturbance level, large temperature gradients, streamwise curvature, possible separations, and

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the transitional Reynolds numbers of the modern gas turbine will provide a severe test of any prediction program. The highly complicated nature of the flow, and the possibility of the interaction of the various phenomena outlined above, discussed in more detail in Daniels [1], make it imperative that any computer program which a turbine designer contemplates using should be adequately tested against experimental data which have been obtained under conditions that simulate turbine operating conditions as closely as possible, such as the data of Daniels [1], and Schultz *et al.* [2].

A survey of the open literature reveals a lack of adequate data. The earliest published work appears to be that of Wilson and Pope [3], which was continued by Walker and Markland [4], their experiments were essentially in incompressible flow, and with a freestream total temperature less than the blade temperature, and hence do not simulate gas turbine practice. Lander [5] and Lander, Fish and Suo [6] measured the heat transfer coefficient distribution to two nozzle guide vanes, in a hot compressible flow produced by an aircraft-type combustion chamber. The heat transfer coefficients were obtained, using a transient calorimeter technique, for ratios of freestream total temperature to blade metal temperature which were not typical of gas turbine practice. Michard [7], Francois *et al.* [8] and Bot, Charpenel and Michard [9], have used similar techniques to Lander *et al.* [5, 6], but their published work contains no blade details or pressure distribution. Turner [10] has published heat transfer rates from a nozzle guide vane at three different turbulence levels. He supplied blade profile and velocity distribution details. A peculiarity of the data is the chordwise fluctuations of the heat transfer coefficients; Daniels [1] discussed the possible reason for this phenomenon. Bayley and Milligan [11] have extended Turner's work [10], Kühl [12] and Köelhler *et al.* [13] use a similar experimental technique to Turner but do not publish any details of the flow or blade profiles. Hanus and Ecuger [14] include blade profiles and flow conditions but only give heat transfer coefficients for a film-cooled blade, as does Louis [15]. Martin, Brown and Garrett [16] do not include blade profiles or flow details and report having considerable data reduction difficulties caused by temperature discontinuities in their instrumented blade surface.

This apparent lack of a suitable data base for testing the boundary layer programs under realistic gas turbine conditions prompted the experimental work reported by Daniels [1] and Schultz [2]. The experiments simulate gas turbine practice in Mach number, Reynolds number and wall to freestream temperature ratio. The turbulence level was as high as possible but is not representative of turbine conditions.

The data of Daniels [1] and Schultz [2] has been used as a test case for a number of computer programs. The results of these predictions and the comparison between them and the experimental data are reported and discussed in this paper.

2. DESCRIPTION OF THE COMPUTER PROGRAMS

2.1. *The Cebeci–Smith program*

This program was used as supplied by its authors. It is a 'mean field closure type'; the fluctuating terms in the momentum and energy equations being specified in the turbulence model in terms of mean flow parameters only. A full description of the program is given in Cebeci and Smith [17]. It is applicable to laminar and turbulent compressible boundary layers in two-dimensional and axisymmetric flows.

The continuity, momentum and energy (total enthalpy in this case) equations, in mass averaged form, are solved in terms of a transformed system of coordinates specified by the Mangler, Levy–Lees transformation. The very efficient Keller's 'Box' numerical method, described by Keller and Cebeci [18], is used. Closure of the equations is achieved by the specification of the fluid bulk properties, assumed to be those of a perfect gas in all the calculations presented here, and by the calculation of the Reynolds stress terms using a mean field turbulent model. An eddy viscosity is calculated using Prandtl's mixing length model with Cebeci and Smith [17] modifications to the Van Driest sublayer damping. This is then related to the Reynolds stresses and the mean velocity gradient in the usual way. Applying Reynolds analogy, in the form of a constant total turbulent Prandtl number, allows a turbulent thermal conductivity to be determined. This enables the fluctuations in total enthalpy to be calculated from the mean total enthalpy gradient. The effects of streamline curvature on the turbulent structure are accounted for in a first order manner by the use of the suggestions of Bradshaw [19].

The use of a gradient diffusion, i.e. eddy viscosity, is known to provide good results where the turbulent boundary layer is self-preserving or local equilibrium conditions apply. Turbulent boundary layers on a turbine blade will obviously not be in a state of self-preservation. However, the inner layer, or log-law region, in nearly all turbulent boundary layers is a local equilibrium region and since approximately 80% of the velocity change in the boundary layer takes place in this region, mixing length and eddy viscosity concepts should provide acceptable predictions unless the pressure gradients are so severe that the log-law breaks down.

In the Cebeci–Smith (C–S) program the location of the transition point is specified as an external input by the user. For the calculations presented here the transition location input was based on the experimental results. The C–S program uses a computational mesh with 200–250 intervals in the x-direction (along the blade surface) and a maximum of 64 steps across the boundary layer. Compile and run time on a CDC 6600 computer is less than a minute per surface.

2.2. *The Patankar–Spalding type program*

This program was a slightly modified version of the well-known Patankar and Spalding Genmix IV program [20]. Turbulence is modelled in a similar way to the C–S program using mixing length and eddy viscosity concepts, but the numerical approach is different. The program was modified by including an experimentally derived turbulence model in an attempt to produce a program more suited to flows over gas turbine blades. Specifically the effects of free-stream turbulence, low Reynolds number flows, streamline curvature and Goertler vorticity on the transition and turbulence structure were incorporated by means of simple correlations, see Forest [21].

The transition location is predicted by means of the correlations developed by Forest [21]. The Patankar–Spalding (P–S) program uses a similar sized grid to the C–S program; both programs using the same input data. The compilation and run time are also similar, a direct comparison is not possible because the P–S program is usually run on an IBM 360 machine.

2.3. *The Cebeci–Smith–McDonald program*

The McDonald turbulence model, McDonald and Fish [22] and Shamroth and McDonald [23], is a one equation model and like the model of Bradshaw and Ferris [24] is based on Townsend's structural hypothesis for equilibrium boundary layers [25–27]. The boundary layer turbulence kinetic energy equation is integrated, the dissipation is modelled using a dissipation length scale and the Reynolds stress is modelled using the mixing length hypothesis. Empirically based formulae are used to input values of the dissipation length scale, the mixing length and a number of structural coefficients that are required in the equation. The dissipation length scale is also modified by factors to account for the effects of wall damping, low Reynolds number flows and streamline curvature, again using Bradshaw's [19] suggestions.

It was possible to incorporate this turbulence model into the C–S program without major changes to the original program. The Cebeci–Smith–McDonald (C–S–M) program attempts to predict transition from the rate of entrainment of the free-stream turbulent energy into the boundary layer. A detailed discussion is in McDonald and Fish [22]. For low free-stream turbulence the C–S and C–S–M programs gave identical results. For high free-stream turbulence the C–S program is obviously not applicable.

The C–S–M program uses an identical grid to the C–S program. All the predictions presented in this paper were run interactively on the OUEL cascade groups DEC PDP 11/34 computer, and running times of 90 min were typical.

2.4. *Wilcox 'EDDYBL' program*

This program is described in detail, with an extensive bibliography, in Wilcox [28]. The program applies to non-similar, laminar, transitional and turbulent compressible boundary-layer flows of perfect gases. It incorporates a two-equation model of turbulence and a number of options for handling the turbulent heat flux including the use of a turbulent heat flux equation. Starting profiles can be used but the self-starting mode was used for the calculations in this report.

The usual boundary layer equations of continuity momentum and energy are used with the turbulent Reynolds shear stress being replaced by $\mu_t(\partial U/\partial Y)$ where μ_t is the turbulent viscosity, U is the local mean velocity in the stream wise direction and Y is the coordinate normal to the blade. The turbulent mixing energy, e, and the turbulence dissipation rate, ω, are introduced by defining $\mu_t = \rho e/\omega$ where ρ is the density. Two non-linear partial differential equations, one for the mixing energy and one for the dissipation rate are set up with e and ω terms appearing in both equations. These are the turbulence model equations. The turbulent heat flux, q_t, can be obtained from a turbulent heat flux equation (incorporating q_t, e and ω) or from $q_t = -k_t(\partial T/\partial Y)$, where k_t is the turbulent conductivity and T is the local mean temperature. k_t is obtained from the turbulent Prandtl number Pr_t and μ_t, while Pr_t may be calculated, input or assumed constant. From a number of trials it was found that there was little difference between the results obtained by using the turbulent heat flux equation or by using a constant Pr_t and so that latter approach was used throughout.

The model predicted transition mechanism of the Wilcox EDDYBL (W–E) program is very similar to that used in the C–S–M program. As the 'laminar' boundary layer develops, turbulent energy is entrained from the freestream, and spreads through the boundary layer by molecular diffusion, so that it increases monotonically from zero at the blade surface to the freestream value at the boundary layer edge. No amplification of the turbulent energy occurs for a significant distance downstream of the leading edge. At some point the turbulent dissipation will be less than the production, and amplification of the entrained freestream turbulent energy takes place.

Details of the Levy–Lees transformation of the equations and the numerical techniques for solving the equations are contained in Wilcox [28]. Approximately 250 steps were used to traverse each blade surface and between 80 and 110 stations were used across the boundary layer. An execution time of about 170 s per surface using an ICL 2980 computer was usual.

2.5. *Wilcox program with all turbulent parameters supplied (W–T)*

Reference has already been made to the provision in the Wilcox program for inputting tabular values of Pr_t. Only small changes were needed to arrange for the program to also allow the inputting of tabular values of μ_t. Using such tabular values allows simplification of the program in that the turbulent model equations do not need to be solved. It was also found that in this case there was no need to keep the iteration feature of the

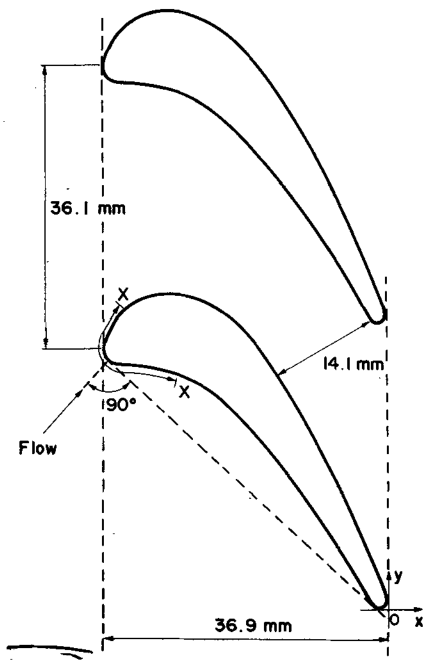

FIG. 1. Blade profile and cascade details.

Table 1. Coordinates of blade tested. Origin of coordinates is 0 in Fig. 1. All dimensions are in mm

x	y	x	y
– 36.694	33.561	– 2.074	0.5771
– 35.269	36.909	– 3.306	2.792
– 33.431	38.693	– 5.144	5.920
– 31.594	39.692	– 6.980	8.868
– 29.757	40.228	– 8.817	11.659
– 27.921	40.383	– 10.654	14.306
– 26.084	40.187	– 12.490	16.826
		– 14.327	19.162
– 24.247	39.667	– 16.164	21.325
– 22.411	38.807	– 18.002	23 341
– 20.572	37.538	– 19.839	25.226
– 18.736	35.857	– 21.675	26.895
– 16.899	33.829	– 23.512	28.221
– 15.062	31.472	– 25.349	29.226
– 13.226	28.736	– 27.185	29.971
– 13.891	25.685	–.29.022	30.484
– 9.552	22.444		
		– 30.860	30.787
– 7.714	18.998	– 32.697	30.920
– 5.877	15.314	– 34.372	30.894
– 4.041	11.348		
– 2.204	7.042		
– 0.368	2.307		

program — a feature introduced because of the way in which non-linear terms of the equations are linearised. Without iteration the results were almost identical to those obtained using five iterations at each step along the surface. Using the same number of steps and stations as in the W–E program execution times were about 35 s per surface using the ICL 2980 computer.

The sets of curves from which the Pr_t and μ_t values were obtained are identical in shape to those used by Browne and Antonia [29]. However, the rate at which the curves develop had to be changed to give reasonable agreement with the experimental results. The curves are plotted against Y^+ where $Y^+ = YU_\tau/\nu$, Y being the distance normal to the surface, U_τ the friction velocity and ν the kinematic viscosity. The rate at which the curves develop is a function of Re_θ, the Reynolds number based on the momentum thickness, and X^+, a dimensionless distance downstream from the start of heating defined by

$$X^+ = \int_0^x U_\tau/\nu, \, \mathrm{d}X,$$

X being the actual distance. The curve developments were different for the suction and pressure surfaces and varied also with the approach Reynolds number and the free-stream turbulence. Thus no generalisations of the curve shapes are as yet possible.

3. DISCUSSION OF RESULTS

3.1. *Experimental conditions*

The blade profile, relevant to the calculations of this paper, is detailed in Fig. 1 and Table 1. The pressure distribution varied very little with Reynolds number as shown in Fig. 2. Two free-stream turbulence levels, one of 0.4% and the other of 4%, were used in the experiments. For calculation purposes two inlet flow rates, see Table 2, were considered. Constant wall and stagnation temperatures of 289 K and 432 K respectively applied to all tests. The wall heat flux measurement technique allowed the test results to be expressed in terms of an isothermal wall. See Daniels [1] for a detailed description and discussion of the transient technique used to obtain the experimental data.

Table 2. Flow conditions for the experiments

Designation	Reynolds number at inlet	Mach number at inlet	Free-stream temperature at inlet, °C	Wall temperature, °C	Total pressure of flow, MPa
Re^-	4.8×10^5	0.38	150	16	0.190
Re^+	1.3×10^6	0.33	150	16	0.581

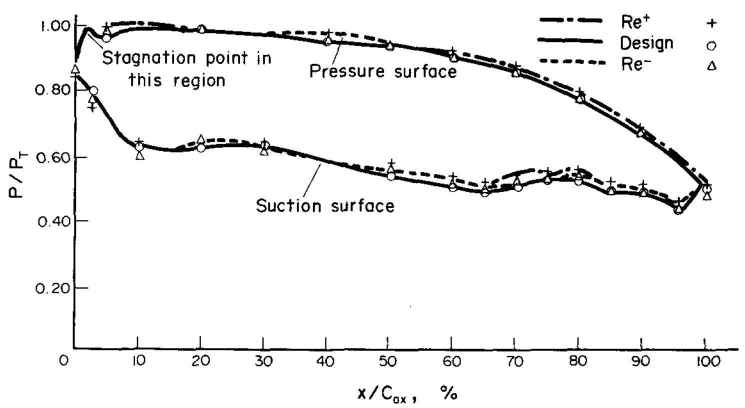

FIG. 2. Experimental pressure distributions along blade surfaces: +—·—+ Re^+ (1.3×10^6); △---△ Re^-(6.73×10^5); ○——○ (6.73×10^5).

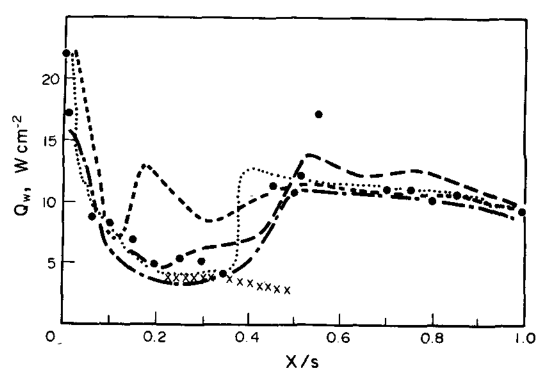

FIG. 3. Wall heat flux distribution: suction surface, Re^-, T_u = 0.4%. —·—·— C–S calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

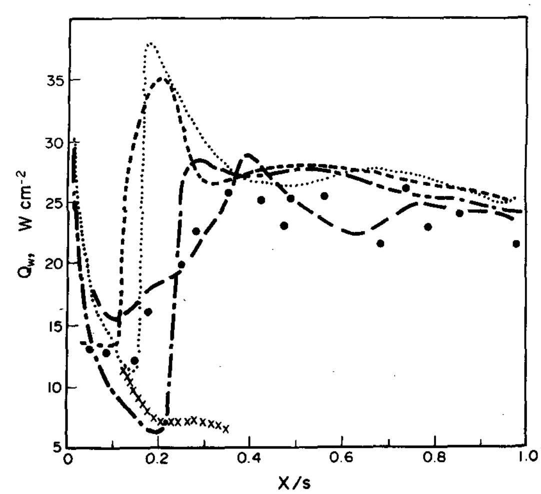

FIG. 5. Wall heat flux distribution: suction surface, Re^+, T_u = 0.4%; —·—·— C–S calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

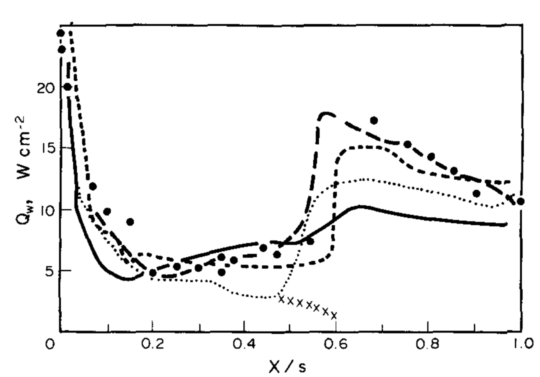

FIG. 4. Wall heat flux distribution: suction surface, Re^-, T_u = 4%; —— C–S–M calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

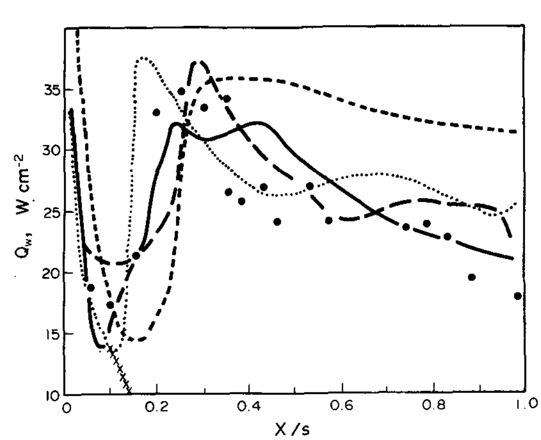

FIG. 6. Wall heat flux distribution: suction surface, Re^+, T_u = 4%; —— C–S–M calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

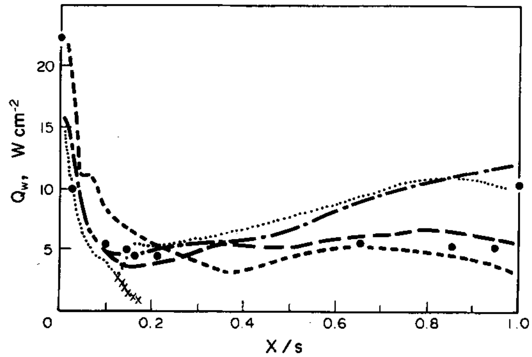

FIG. 7. Wall heat flux distribution: pressure surface, Re^-, T_u = 0.4%; —·—·— C–S calculations; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

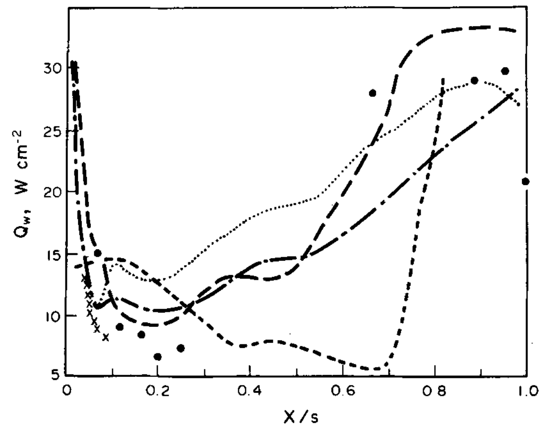

FIG. 9. Wall heat flux distribution: pressure surface, Re^+, T_u = 0.4%; —·—·— C–S calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

3.2. *Comparison between computed and measured values*

An extensive discussion of the experimental results is contained in Daniels [1]. Here we are concerned with the performance of the computer programs. Representative calculations compared with the experimental results are shown in Figs. 3 and 10. Figures 3–6 are for the suction surface while Figs. 7–10 are for the pressure surface.

3.2.1. *Suction surface.* With the exception of the P–S program in the low Reynolds number, low turbulence flow of Fig. 3, all the programs produce predictions which behave qualitatively in the same way as the experimental results, i.e. a high leading edge heat transfer rate, followed by a rapid fall in the laminar section of the boundary layer, a sharp rise in the transition region, followed by a gradually falling heat transfer rate towards the trailing edge as the turbulent boundary layer develops, Figs. 3–6.

Quantitatively there are significant differences between the calculated values and the experimental data. The uncertainty in the experimental data is considered to be of the order of 10%, and on parts of the suction surface the predictions differ from the data by amounts much greater than this.

In the low Reynolds number flow of Fig. 3, the lower turbulence case, predictions and experimental data agree over the laminar and turbulent sections of the blade boundary layer, however, there are discrepancies in the transition region (0.2–0.6 X/S). The P–S calculation exhibits a rather strange transition under these conditions, and the W–T program overestimates the level of turbulent heat transfer in the region downstream from the transition point.

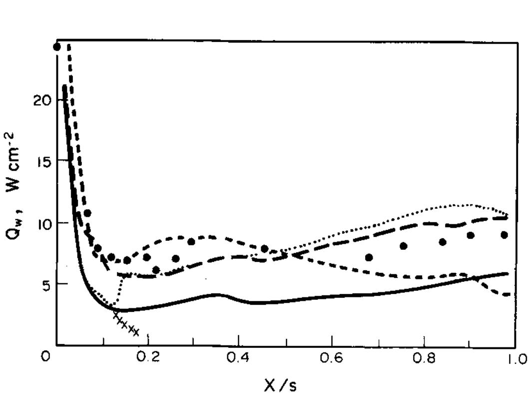

FIG. 8. Wall heat flux distribution: pressure surface Re^-, T_u = 4%; —— C–S–M calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

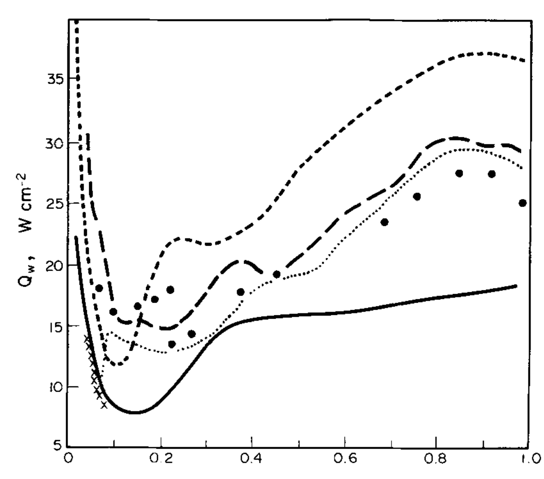

FIG. 10. Wall heat flux distribution: pressure surface, Re^+, T_u = 4%; —— C–AS–M calculation; ---- P–S calculation; ···· W–E calculation; ——— W–T calculation; × × × laminar flow calculation using W–E; ● experimental results.

At the higher turbulence level, all the programs tend to underestimate the levels of the heat transfer in the laminar region, i.e. up to about $0.2\,X/S$. This is probably due to the lack of, or the poor modelling of, the enhancement of the laminar heat transfer rates due to freestream turbulence. Daniels [1] gives a discussion of this phenomenon. In the region of the turbulent boundary layer i.e. on the blade 'back surface' 0.6–1.0 X/S of the blade, with the exception of the W–T program, all the predictions underestimate the heat transfer rate. The transition region as in the low turbulence case causes problems, with the exception of the W–E program the predictions are quantitatively within 10–15% of the experimental data, but qualitatively the predictions and the experiments do not follow quite the same trends. It must be pointed out that the experimental data shows a curious behaviour in that at this low Reynolds number condition the effect of raising the turbulence level has been to move the transition from $0.5\,X/S$ in Fig. 3 to $0.6\,X/S$ in Fig. 4. See Daniels [1] for a discussion of this phenomenon.

In the high Reynolds number flows of Figs. 5 and 6 the predictions for the 'back surface' heat flux are within the experimental tolerance on the data, and quantitatively seem to follow the trend, the exception being the P–S program in the high turbulence case. Again as with the low Reynolds number cases, agreement in the transition region is poor.

The C–S–M program prediction for the high Reynolds number high turbulence case of Fig. 6 are in good agreement with the data.

Overall it does not appear that any one of the programs produces predictions which are any better in their agreement with the experimental data than any other, and no obvious advantages seem to accrue from the use of the more complex turbulence models as used by the C–S–M or W–E programs. The fact that, with the previous noted exception of the P–S program in Fig. 7, all the programs agree with the experimental data on the blade 'back surface' 0.6–1.0 X/S, is only to be expected since in this region the boundary layer is turbulent and developing beneath a virtually constant velocity mainstream.

3.2.2. *Pressure surface.* Here the boundary layer development is expected to be far more complicated than on the suction surface; the concave curvature and strongly acceleration pressure gradients producing contradictory effects on the boundary layer development; the concave curvature promoting transition and possibly leading to the formation of Goertler type vorticity, Kemp [30], Cox [31], whilst the accelerating mainstream will tend to delay transition and damp the turbulence generating mechanisms. For a given pressure gradient, the 'strength' of the turbulence damping mechanisms as measured by the Launder and Jones [32] relaminarisation parameter are inversely proportional to the Reynolds number, whilst the generation of 'Goertler' vorticity as measured by the Goertler number is proportional to the Reynolds number. Strictly the Reynolds number based the boundary layer momentum thickness.

The marked change in the surface heat flux as the Reynolds number is increased as shown in Figs. 7 and 8, the low Reynolds number cases, to Figs. 8 and 9, the high Reynolds number cases, can possibly be attributed to the effect of the two phenomena outlined above.

The complexity of the pressure surface boundary layer development is reflected in the very poor quantitative and qualitative agreement between the experimental data and the computer prediction. Apart from the P–S and W–T predictions for the low Reynolds number low turbulence case of Fig. 7 and the W–E and W–T predictions for the high Reynolds number high turbulence case of Fig. 10, agreement between predictions and experimental data particularly in the crucial trailing edge region $0.4\,X/S$ onwards is very poor.

4. CONCLUSIONS

Five computer programs each with a different turbulence model, the turbulence models varying in complexity from a simple running length model to a 'two equation' model, have been used to predict the surface heat flux to a gas turbine rotor blade cascade operating at conditions which were a closer representation of gas turbine practice than has previously been obtainable.

Comparisons between the experimental data and the predictions showed with certain exceptions good agreement for the laminar leading edge region and for the fully turbulent region on the suction surface. The transition region on the suction surface was not well predicted. On the pressure surface the agreement between the predictions and the data was generally poor.

From the limited amount of data presented no advantages seem to accrue from the use of the more complicated turbulence models for the prediction of the suction surface boundary layer, the major difficulties being in the prediction of the transition and the effect of the freestream turbulence on the laminar boundary layer. On the pressure surface the use of the more complex two equation turbulence model, i.e. the W–E program, produced a better qualitative indication of the boundary layer behaviour although the quantitative agreement was very poor.

Computationally the more complex models e.g. C–S–M and W–E took much longer to run per surface and were difficult to modify. It is not yet clear if, in light of these facts and the experience outlined in this paper, their use as a design tool can be justified.

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CALCUL DES FLUX DE TRANSFERT THERMIQUE SUR LES AUBES DE TURBINES A GAZ

Résumé—On utilise cinq programmes de calcul pour évaluer le flux de chaleur sur les aubes de turbine à gaz et les résultats sont comparés à ceux d'un travail expérimental sur des grilles d'aubes. Les programmes concernent différentes méthodes pour modéliser les grandeurs caractéristiques de la turbulence. L'étude expérimentale a été conduite récemment dans la soufflerie à piston libre de l'Université d'Oxford.

BERECHNUNG DES WÄRMEÜBERGANGS AN GASTURBINENSCHAUFELN

Zusammenfassung — Fünf Computerprogramme wurden benutzt, um den Wärmeübergang an Gasturbinenschaufeln zu berechnen. Die Ergebnisse wurden dann mit experimentellen Werten von Schaufelgittern verglichen. In den Programmen werden verschiedene Methoden bei der Modellierung der Turbulenzgrößen verwendet. Die experimentellen Untersuchungen wurden im Freikolbenkanal der Universität von Oxford durchgeführt.

РАСЧЁТ СКОРОСТИ ПЕРЕНОСА ТЕПЛА К ЛОПАТКАМ ГАЗОВОЙ ТУРБИНЫ

Аннотация — Для расчёта скоростей переноса тепла к лопаткам газовой турбины используются пять вычислительных программ, результаты счёта по которым затем сравниваются с данными эксперимента на дисках лопаток. В программах применяются различные варианты моделирования вклада турбулентности. Экспериментальная работа выполнена в Оксфордском Университете на аэродинамической трубе с открытой рабочей частью.